

EECS 360 Lab 11

Laplace Transform

The Laplace transform of a signal $x(t)$,

$$x(t) \xrightarrow{L} X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt$$

is a generalization of the continuous-time Fourier transform that is useful for studying CT signals and system. When $s = j\omega$, i.e. the Laplace transform reduces to the CTFT.

Most of the times, the Laplace transform can be represented as a ratio of polynomials in s :

$$X(s) = \frac{N(s)}{D(s)}$$

which is also known as rational transforms. Rational transforms can be completely determined by the roots of the polynomial $N(s)$ and $D(s)$, known as zeros and poles.

1. pole-zero diagram.

A pole-zero diagram displays the “poles” and “zeros” of the rational transform by placing an ‘x’ at each pole location and an ‘o’ at each zero location in the complex s -plane.

Poles and zeros can be found out by using *roots* function in matlab, i.e.

Transfer function:

$$H(s) = \frac{s - 1}{s^2 + 3s + 2}$$

```
>>b = [1 -1]; % Numerator coefficients
>>a = [1 3 2]; % Demoninator coefficients
>>zs = roots(b)
zs =
    1
>>ps = roots(a)
ps =
   -2
   -1
>>pzmap(ps,zs) % generates pole-zero diagram
```

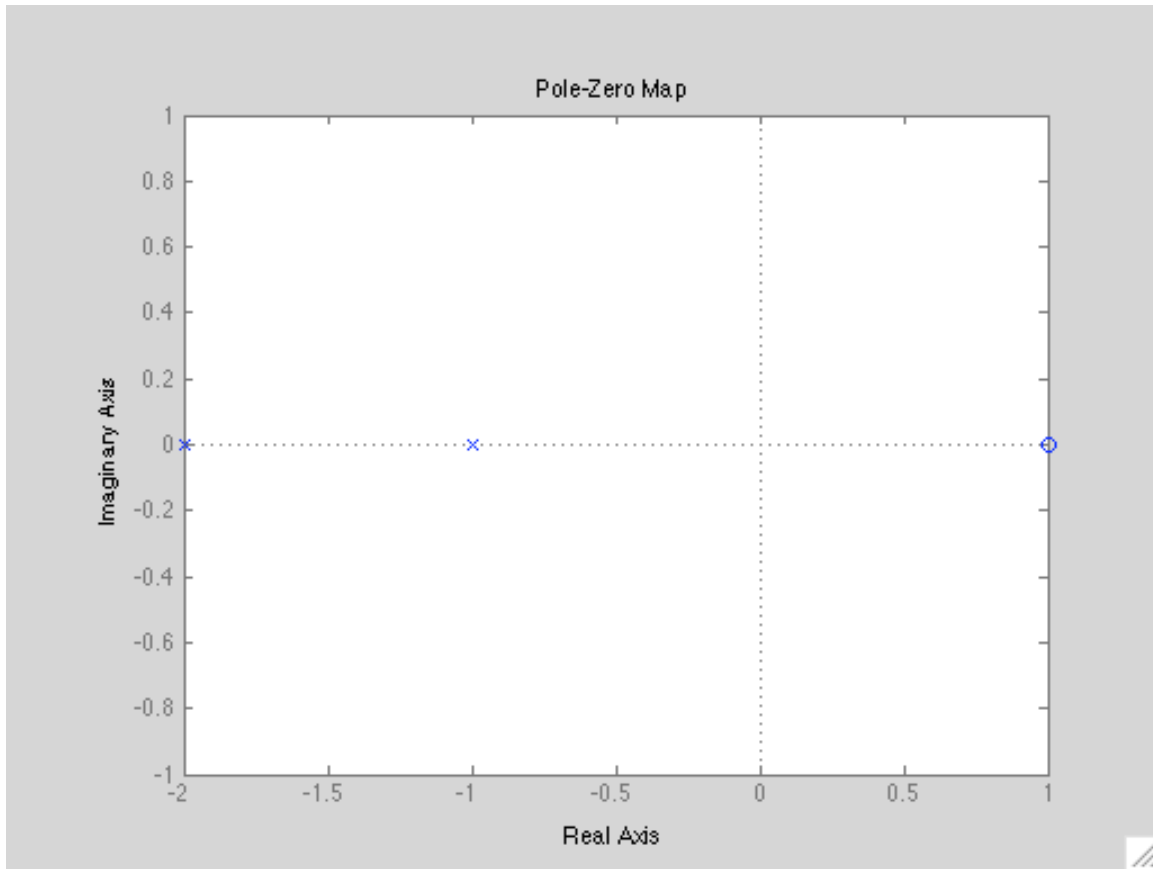


Fig.1 Pole-zero diagram

Using the method given above, find out the zeros and poles of the following system functions and plot them:

$$(1). H(s) = \frac{s + 5}{s^2 + 2s + 3}$$

$$(2). H(s) = \frac{2s^2 + 5s + 12}{s^2 + 2s + 10}$$

$$(3). H(s) = \frac{2s^2 + 5s + 12}{(s^2 + 2s + 10)(s + 2)}$$

2. Surface plots of Laplace transforms

Consider a transfer function:

$$H(s) = \frac{s^2 + 2s + 17}{s^2 + 4s + 104}$$

(1). Define the numerator and denominator polynomial coefficients as vector **b** and **a** respectively.

(2). Use the *freqs* function to evaluate the frequency response of a Laplace transform.

```
H = freqs(b, a, omega);
```

where $-20 \leq \omega \leq 20$ (*omega*) is the frequency vector in rad/s. (Hint: use *linspace* to generate a vector with 200 samples.)

(3). Graph the magnitude and phase of the frequency response.

(4). Complex number s in the Laplace transform is represented as:

$$s = \sigma + j\omega$$

A 3-D surface plot of the system transform function $H(s)$ at the range of interests, i.e. $-20 \leq \omega \leq 20$ and, $-5 \leq \sigma \leq 5$ is extremely useful to illustrate the relationship between the frequency response $H(s)$ and the pole-zero locations.

the system response matrix s can be generated from ω and σ using *meshgrid* function:

```
[sigmagrid, omegagrid] = meshgrid(sigma, omega);
```

hence, $s = \sigma + j\omega$ is:

```
sgrid = sigmagrid + j*omegagrid;
```

use function *polyval* to evaluate the numerator and denominator polynomials at the specific range:

```
H1 = polyval(b, sgrid) ./ polyval(a, sgrid);
```

Finally, use *mesh()* function to generate the surface graph of the magnitude of $H(s)$ in dB:

```
mesh(sigma, omega, 10*log10(abs(H1)))
```

Questions:

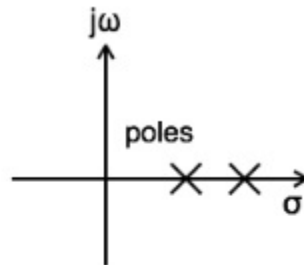
1. What do you understand by pole and zero of transfer function of a system?
2. Can we have a pole and a zero at the same point of the graph? Give an example of such transfer function.
3. Can we have multiple poles or zeros at the same point of the graph? Give an example of such transfer function.
4. Can we decide about stability and causality from pole zero curve?
5. Find out the poles and zeros in your surface plot.

6. Comment on Causality and stability of the system based on your pole zero graph.

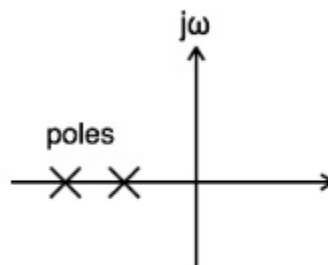
Notes

Causality and Stability

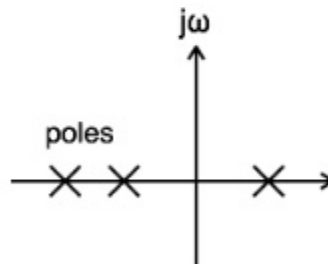
- For a system to be causal, all poles of its transfer function must be right half of s-plane.



- A system is said to be stable when all poles of its transfer function lay on the left half of s-plane.



- A system is said to be unstable when at least one pole of its transfer function is shifted to the right half of s-plane.



- A system is said to be marginally stable when at least one pole of its transfer function lies on the $j\omega$ axis of s-plane.

